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PARTIAL-WAVE ANALYSES OF THE $(3\pi)^-$ SYSTEM IN THE REACTION $\pi^- p \to \pi^- \pi^- \pi^+ p$ AT 11.2 GeV/c

Bologna-Florence-Genoa-Milan-Oxford-Pavia Collaboration

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The results are presented of two partial-wave analyses of the $(3\pi)^-$ system in 30 000 events of the reaction $\pi^- p \to \pi^- \pi^- \pi^+ p$ at 11.2 GeV/c. Both techniques incorporate the assumptions of the isobar model and are (a) the University of Illinois program which fits in terms of the (3π) density matrix elements and (b) an amplitude parametrisation including possible effects of both spin non-flip and spin flip at the baryon vertex. The results obtained with these independent programs are found to be very close.

The proportions of contributing states, their t-dependence and the phases of off-diagonal density matrix elements are discussed for the mass range $0.9 < M(3\pi) < 1.9$ GeV/c^2. The A_1 phenomenon is investigated in the context of a recent model; values of the mass, width and cross sections for the A_1, A_2, A_3 states are given, and an interesting variation in the $2^+P_{M=1}$ wave is observed at high masses.

1. Introduction

Considerable interest [1] has recently been aroused in three-body partial-wave analyses and already much important information has been gained. Most of these analyses employ the program written at the University of Illinois [2] whilst others analyse in terms of amplitudes [3]. This paper contains a comparison of results from what has become the standard density matrix program with those from an amplitude parametrisation.

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Although there are now a number of three-pion analyses a considerable degree of controversy still surrounds the results. The A_2 has been confirmed as a *bona fide* resonance both in intensity and phase variation. The A_1 phenomenon has a phase change which is too small to be regarded as due to a simple resonance alone, but a recent paper [4] attempts to save the 1^+ SU(3) octet by claiming that interaction with a Deck-like effect [5] will reduce the expected phase variation. In the high mass region this experiment confirms recent observations of a phase variation in the 2^-S ($f\pi$) wave in the $(3\pi)^+$ system [6, 7] which at first seemed to contradict the results in the $(3\pi)^-$ system [8], whilst interesting variations of the $2^+P_{M=1}$ ($f\pi$) wave are noted.

2. Experimental data

The source of the data for this analysis is an 11.2 GeV/c π^-p experiment performed in the CERN 2 m chamber by the Universities of Bologna, Florence, Genoa, Milan, Oxford and Pavia. 500 000 pictures were taken in April 1971 and 350 000 in June 1972.

The sample used in this study derives from the 29 611 events which had a fit to the hypothesis

$$\pi^-p \to \pi^-\pi^-\pi^+p\,,$$

with an acceptable kinematic χ^2 probability and consistent with ionisation requirements. The beam momentum bite taken for the analysis was 11.19 ± 0.20 GeV/c. Some 40% of the film has been measured on the Oxford University PEPR system and reconstructed using the RHEL geometry and kinematics programs. The rest of the film was measured on HPD's at the CNAF in Bologna and was processed through the THRESH-GRIND program chain. Technical details of the experimental procedures can be found in ref. [9]*. The resolution on the 3π mass is between 10 and 12 MeV/c^2 in the mass region of interest, and that on 2π mass is between 6 and 10 MeV/c^2.

The three-pion mass spectrum can be seen in fig. 1, the shaded region corresponding to events with $1.12 < M(p\pi^+) < 1.32$ GeV/c^2 that is reflection of Δ^{++} production. It can be seen that contamination from this source is only serious at higher (3π) masses but for use in the Illinois program, events in this region were anti-selected throughout, the corresponding phase-space normalization integrals being corrected accordingly. The possible reflection of Δ^0 and N*'s was also tested and cuts of varying severity were tried on the $(p\pi^-)$ and $(p\pi^+)$ spectra. It may be asserted that none of the results quoted would shift by more than a small fraction of a standard deviation with bigger cuts than those used, and for the final analysis no $(p\pi^-)$ cuts

* For details of the experimental procedures and the amplitude analysis, see ref. [9a]; for details of data compatibility, contamination checks and the use of the Illinois program, see ref. [9b].

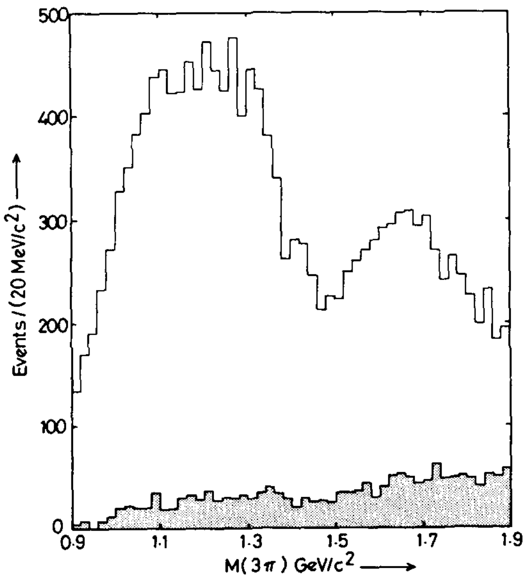

Fig. 1. The effective mass spectrum of ($\pi^-\pi^-\pi^+$). The shaded region corresponds to events with $1.12 < M(p\pi^+) < 1.32$ GeV/c^2.

at all were found to be necessary. Spectra for the ($p\pi\pi$) systems are reproduced in Monte-Carlo event generation using the fitted (3π) density matrix elements, indicating that three-body decays of N*'s or Δ's are not important.

3. The isobar model

Both methods in the analysis use a Bose-symmetrised isobar model (the terminol-

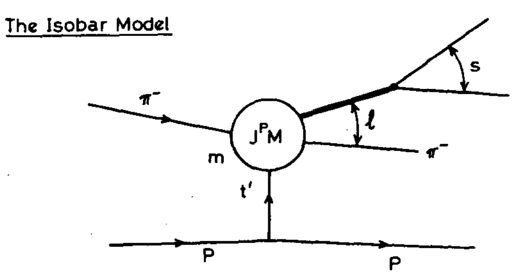

Fig. 2. The terminology of the isobar model used. A (3π) mass m is produced with momentum transfer $t' = t - t_{min}$ with spin-parity J^P z-component M. It decays *via* an l-wave into an isobar of spin s ($J = l + s$) and an odd π^-.

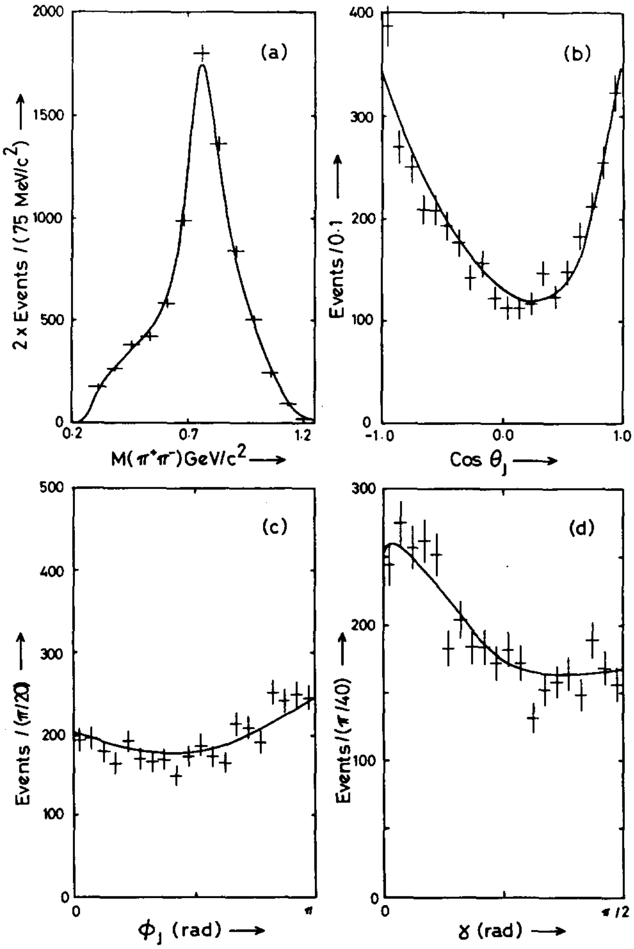

Fig. 3. Distributions from the interval $1.2 < M(3\pi) < 1.4$ GeV/c^2 and $0 < |t'| < 0.5$ (GeV/c)^2 with a fit to the data; (a) mass ($\pi^+\pi^-$), two entries per event; (b) cos θ_J in the Jackson frame; (c) ϕ_J in the Jackson frame; (d) γ, the third Euler angle defining the (3π) system, is the rotation angle of the $\pi^-\pi^-$ plane about the π^+ direction in the (3π) rest frame.

ogy of which is illustrated in fig. 2), with a maximum likelihood optimization in which the 3π system is fully represented by the required $3n - 4 = 5$ kinematic variables. Two of these are the neutral dipion masses squared, but to make the approaches as different as possible within these constraints, the usual Gottfried-Jackson frame cos θ, ϕ of the π^+, and the $\pi^-\pi^-$ plane rotation angle γ were replaced in the amplitude analysis by the momentum vectors of the final state particles.

In fig. 3 is shown typical distributions in the density matrix analysis variables for the 3π mass interval 1.2 to 1.4 GeV/c^2 and $0.0 \leqslant |t'| \leqslant 0.5$ (GeV/c)^2. The

Table 1
Masses and widths of isobars

Dipion spin	Mass (GeV/c^2)	Width (GeV/c^2)
0 (ϵ)	0.747 ± 0.035	$0.360^{+0.090}_{-0.081}$
1 (ρ)	0.766 ± 0.005	0.137 ± 0.012
2 (f)	1.272 ± 0.010	0.152 ± 0.020

smooth curves here represent the results of one fit, using the Illinois program.

A full account of the Illinois method can be found in ref. [2] and of the amplitude parametrisation in ref. [9a] where there is also a detailed comparison of the two.

Both programs use as decay states those combinations of (3π) spin states which have a well-defined Y-parity (η); states of $\eta = +1(-1)$ can be shown to correspond to natural (unnatural) parity exchange in a t-channel model in the limit of small $1/s$ [10]. It can be shown that states of different η do not interfere in the absence of polarisation information. In practice it was found that production of the $\pi^-\pi^-\pi^+$ system is dominantly in the $\eta = +1$ and $M = 0$ states. For this reason these quantum numbers are assumed in the naming of states in this paper, unless noted otherwise.

The dominant ($\pi^+\pi^-$) states are those of the ϵ^0, ρ^0 and f^0 and were included in each decay amplitude by the relevant relativistic Breit-Wigner propagator. No attempt has yet been made to include dipion spin $\geqslant 3$. In the case of the ϵ^0, this should be thought of only as a convenient parametrisation of the spin-zero phase shift, and in order to test the sensitivity of the fitted results to this a direct representation in terms of experimentally measured phase shifts [11] was substituted with no appreciable change in results.

The values used for the masses and widths of the isobars were taken from separate runs of the Illinois program as those which maximised the overall likelihood. The best choice is given in table 1, with typical error bars that result in a change of log likelihood of 0.5.

4. The different parametrisations

In the Illinois program the production mechanism is expressed in terms of a density matrix requiring $\sim n^2$ parameters for n partial waves (not allowing for completely coherent or incoherent production processes) whereas if the production process is parametrised solely in terms of amplitudes, only $\sim 4n$ parameters are needed to specify the real and imaginary parts of the amplitudes corresponding to proton spin flip and proton spin non-flip. Despite this nomenclature it should be stressed that no information is obtained on spin orientations at the baryon vertex, since there is an essential ambiguity such that these amplitudes may be rotated at random in the proton spin-space. In effect, an observable density matrix element is

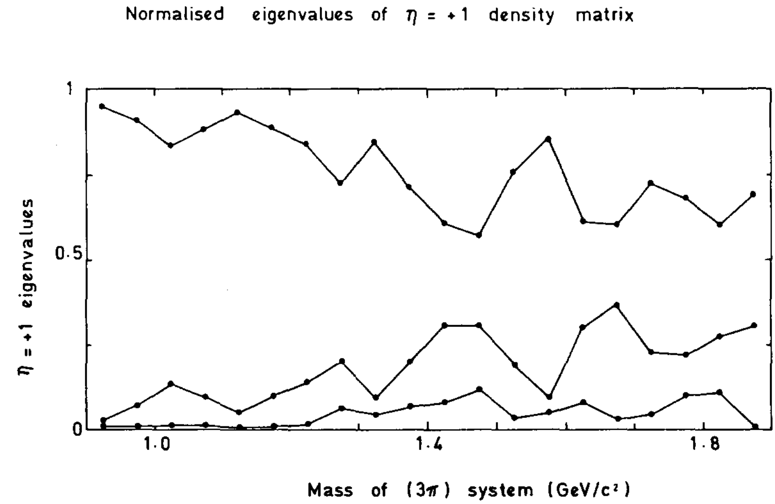

Fig. 4. The normalized three largest eigenvalues of the $\eta = +1$ density matrix as a function of three-pion mass.

constructed between states i and j having the form

$$\rho_{ij}^{\eta} = F_{++}^{i\eta} F_{++}^{j\eta *} + F_{+-}^{i\eta} F_{+-}^{j\eta *} \, ,$$

where the subscripts to the amplitudes denote the proton spin state before and after the interactions.

This implies a physical limitation of rank 2 (for each η) to the density matrix, i.e. only two non-zero eigenvalues. Should incoherence from any source other than the spin history of the proton be present (for example because of differing 3π mass (m) and momentum transfer ($t' = t - t_{\min}$) behaviour of the interfering partial waves), a density matrix formalism is able to parametrise this by an extension of the rank. Similarly, it is possible that an amplitude parametrisation may find the necessity for "spin-flip" terms merely to deal with this problem. In fig. 4 the comparative sizes of the dominant $\eta = +1$ density matrix eigenvalues are plotted as a function of m.

The production process is a function at least of (m, t') but limitations of statistics dictate that for presently available data, mass bins ~50 MeV/c^2 must be chosen with a wide t' bin ($0.0 < |t'| < 0.5$ GeV/c)^2) for an energy-independent analysis.

5. A comparison of the results

This section presents the detailed results of the comparison of the amplitude and density matrix programs in the mass bin $1.20 < M(3\pi) < 1.25$ GeV/c^2 where, as

Table 2
Comparison of amplitude analysis and density matrix results

(a)	Partial wave	Number of events	
		Amplitude program	Illinois program
	0^-S	110 ± 27	79 ± 28
	0^-P	41 ± 21	69 ± 25
	1^+S	469 ± 46	446 ± 48
	1^+P	186 ± 23	129 ± 29
	$2^+D_{M=1}$	64 ± 13	66 ± 19
	2^-P	38 ± 12	50 ± 26
(b)	Partial waves	Phase of density matrix element (deg)	
		Amplitude program	Illinois program
	$1^+S/1^+P$	−76 ± 5	−86 ± 8
	$1^+S/0^-S$	−159 ± 12	−163 ± 13
	$2^+D/1^+S$	−23 ± 5	−19 ± 11
	$2^+D/0^-S$	−173 ± 44	−163 ± 39
(c)	Partial waves	Coherence factor	
		Amplitude program	Illinois program
	$1^+S/1^+P$	0.961 ± 0.091	1 (assumption)
	$1^+S/0^-S$	0.511 ± 0.103	0.623 ± 0.176
	$2^+D/1^+S$	0.935 ± 0.234	0.915 ± 0.220
	$2^+D/0^-S$	0.753 ± 0.222	0.554 ± 0.255

seen in fig. 4, the rank of the matrix is still clearly two. The two sets of results are, however, broadly compatible for all other (3π) mass bins. The integrated intensitites (i.e. the number of events) and the relative phases associated with the dominant partial waves are compared. The amplitude program incorporated degrees of freedom corresponding to both spin non-flip and spin flip for every partial wave.

In table 2(a) is shown the number of events in each of the six dominant partial waves at this energy. It should be observed that the sum total is not quite the same for both programs; this is because of interference and also a small background contribution, to the exact combination of which the main partial waves are insensitive. The agreement between the two sets of results is seen to be good.

Comparison of the phases between partial waves is complicated by both the need for the two phases, one between "spin non-flip" amplitudes and one between "spin flip", and further by an overall rotational ambiguity, as previously explained. An unambiguous measure of the interference between two partial waves is given by the corresponding off-diagonal density matrix element, and the phases of these are given in table 2(b) for the 1^+S and 2^+D with respect to their strongest background waves. Although phases tend to be less well-determined than magnitudes of amplitudes, it is seen that there is still excellent agreement.

In order to complete the comparison of the off-diagonal density matrix elements, the coherence factors between the same pairs of states as used for the phase comparison are given in table 2(c). The coherence factor between two states i and j is defined as

$$\alpha_{ij} = \frac{|\rho_{ij}|}{\sqrt{\rho_{ii}\rho_{jj}}} ,$$

where ρ_{ij} is the corresponding density matrix element. For $\alpha < 1$ there is a possible maximum systematic difference between the phase of the off-diagonal density matrix element and the corresponding dominant amplitude phase difference of $\cos^{-1}\alpha$, if the incoherence has been introduced truly by the presence of spin-flip terms. The agreement between the two sets of results is again seen to be entirely satisfactory.

It may therefore be concluded that the two approaches are in excellent agreement and that an independent check has verified the results of the much used University of Illinois program.

6. Mass variation of the partial waves

Using the density matrix program the intensity of each contributing partial wave and the phase of the off-diagonal density matrix elements has been extracted in 50 MeV/c^2 wide mass bins in the t' interval $0.0 < |t'| < 0.5$ (GeV/c)^2 from 0.9 to 1.9 GeV/c^2. For each fit a dependence of all density matrix elements e^{At} was presumed where A was found as a function of 3π mass from separate fits performed independently. The partial waves required to adequately fit the data in the mass region are detailed in table 3. In any given mass bin some of these waves have negligible contributions and it was found possible to use the assumption of coherence* between different two-body decay modes of the same J^P state to limit the numbei of parameters to <50 throughout, with a bin occupation of $\gtrsim 400$ events. There was, essentially, no problem from ambiguities between different hypotheses of contributing partial-wave sets, but there is less confidence in the last few data points where it is possible that $J = 4$ waves may be entering.

The results of this global analysis may be seen in figs. 5 and 6, which have a close resemblance to the other analyses done in this area, i.e. dominance of the low mass region by the 1^+S ($\rho\pi$) structure, a clear A_2 peak in $2^+D_{M=1}$, and many waves contributing at high mass. Further subdivisions of the data and detailed discussion follow in the succeeding sections.

* I.e. the decay matrix element $\mathcal{M}^{J^PM\eta} = \Sigma_l C_l^{J^P} \mathcal{M}^{J^PMl\eta}$.

Table 3
Partial waves required

	J	P	l	M	η	Decay
1			Flat [a]			3π
2	0	−	S	0	+	$\epsilon\pi$
3	0	−	P	0	+	$\rho\pi$
4	1	+	P	0	+	$\epsilon\pi$
5	1	+	P	1	+	$\epsilon\pi$
6	1	+	S	0	+	$\rho\pi$
7	1	+	S	1	+	$\rho\pi$
8	2	−	S	0	+	$f\pi$
9	2	−	P	0	+	$\rho\pi$
10	2	−	D	0	+	$\epsilon\pi$
11	2	+	D	1	+	$\rho\pi$
12 [b]	2	+	D	2	+	$\rho\pi$
13	2	+	P	1	+	$f\pi$
14	3	+	P	0	+	$f\pi$
15	3	+	D	0	+	$\rho\pi$
16	3	+	F	0	+	$\epsilon\pi$
17 [b]	2	+	D	0	−	$\rho\pi$
18 [b]	2	+	D	1	−	$\rho\pi$
19 [b]	2	+	D	2	−	$\rho\pi$

[a] An incoherent amplitude, isotropic in all angular distributions and populating the Dalitz plot evenly.

[b] These waves were required only for the evaluation of the A_2 density matrix.

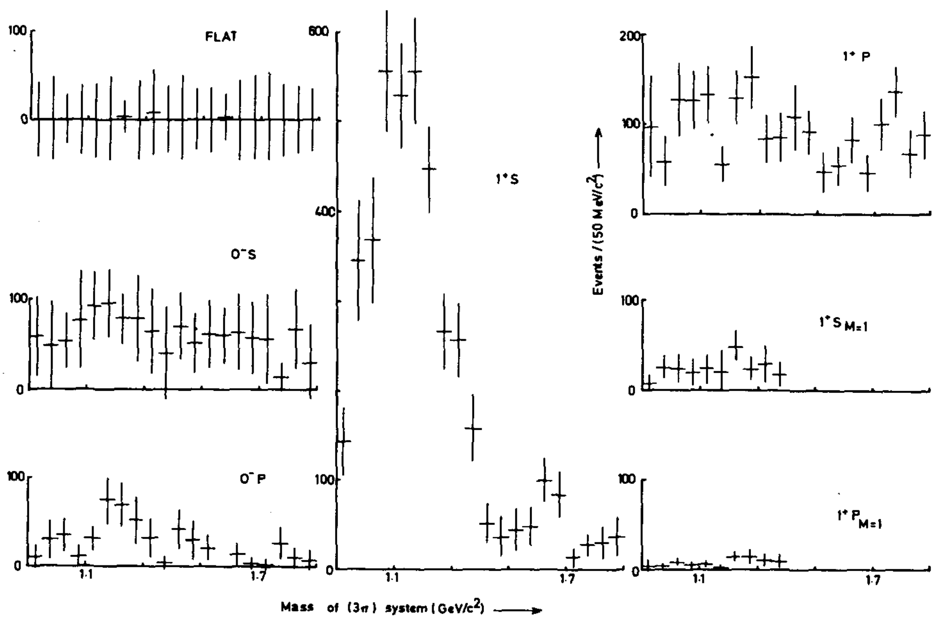

Fig. 5. The low spin partial-wave intensities as a function of 3π mass.

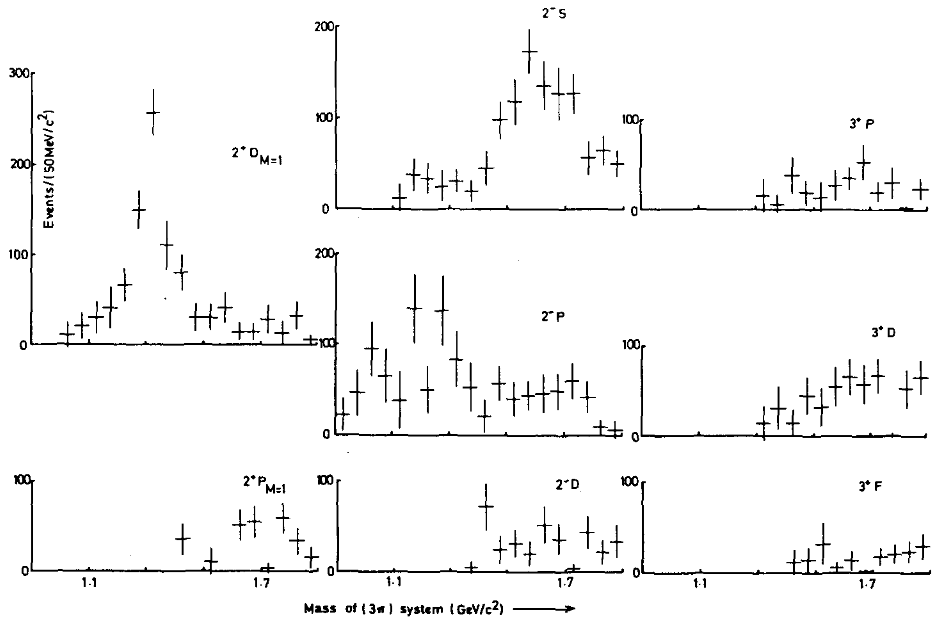

Fig. 6. The high spin partial-wave intensities as a function of 3π mass.

7. The A_1 phenomenon

The enhancement in the $\rho\pi$ 1^+ spectrum at low mass has been the subject of controversy for some time. Whilst it is possible to put a Breit-Wigner through our experimental points the expected phase change of 90° is in reality only ~40° using as comparison wave 0^-S ($\epsilon\pi$) whose intensity is constant through the region. Ascoli et al. [5] have published results from a reggeised Deck model which gives a good prediction for all partial waves below 1.5 GeV/c^2, but if the concept of any resonance in this area is to be relinquished the 1^+ SU(3) octet is in danger.

Bowler at al. [4] have shown that it is possible to make good some small imperfections of the Ascoli model, e.g. a rather smaller 1^+S intensity relative to 0^-S than the data shows, by insertion of a diffractively produced resonance at a mass ~1.3 GeV/c^2. In what is essentially a heuristic formalism to represent a pure Deck mechanism, a pure resonance and rescattering of the Deck $\rho\pi$ system through the resonance, they show that it is possible to represent the 1^+S intensity very well while simultaneously depressing most of the phase change expected for the resonance. The model formula is given in table 4.

This model gives a good parametrization of existing 1^+S partial-wave analysis spectra; indeed, with two exceptions, the same parameters as used in ref. [4] to fit the high statistics data of Antipov et al. [8] also fit the intensity and phase obtained

Table 4
Fit of model formula[a] to A_1 1^+S wave

Experiment	t region $(GeV/c)^2$	τ/p_0	χ	$\alpha(GeV/c)^{-3/2}$	Q	$M_0(GeV/c^2)$	$\Gamma(GeV/c^2)$	χ^2/NDF[b]
40 GeV/c [4]	0.04 – 0.3 0.1 – 0.3 0.17 – 0.3	0.086	−212°	2.24	28°	1.280	0.178	
this exp.	0 – 0.5 (t')	0.055 ± 0.005	−176° ± 3°					20/18
	0 – 0.1 (t')	0.044 ± 0.005	−194° ± 2°					6/16
	0.1 – 0.6 (t')	0.069 ± 0.005	−148° ± 2°			Fixed as above		8/16
	0 – 0.1 0.1 – 0.6 (t') simultaneous fit	0.055 ± 0.005	−178° ± 5°					44/41

[a] Bowler model formula [4] for 1^+S ($\rho\pi$) amplitude:

$$F = \frac{P(M_0 - M) + \tau}{M_0 - M - \frac{1}{2}i\Gamma - k/k_0}\, e^{i\chi}, \qquad P = P_0\, e^{-\alpha\Delta^{3/2}}\, e^{i(0.5 + (\Delta/0.6)0.87)},$$

with Δ the difference between M and its value at threshold where M is the $\rho\pi$ mass, k the $\rho\pi$ phase-space factor ($k = k_0$ at $M = M_0$).

[b] The fits are over the interval $0.9 < M < 1.5$ GeV/c^2.

in this experiment. The results of all fits are given in table 4. The two parameters, which it is necessary to alter, are those governing the relative amount of resonant and Deck amplitudes (τ/p_0) and the overall reference phase χ of no dynamical content.

These differences may be attributed to the t' intervals over which the partial-wave analysis is performed in the two experiments (see table 4). As shown in fig. 7 a and b and noted by Tabak et al. [12] the intensity and phase of the 1^+ S amplitude depend on t': the high t' ($0.1 \leqslant |t'| \leqslant 0.6$ (GeV/c)^2) mass spectrum peaks some 100 MeV/c^2 above the low t' ($0 \leqslant |t'| \leqslant 0.1$ (GeV/c)^2) value and the phase

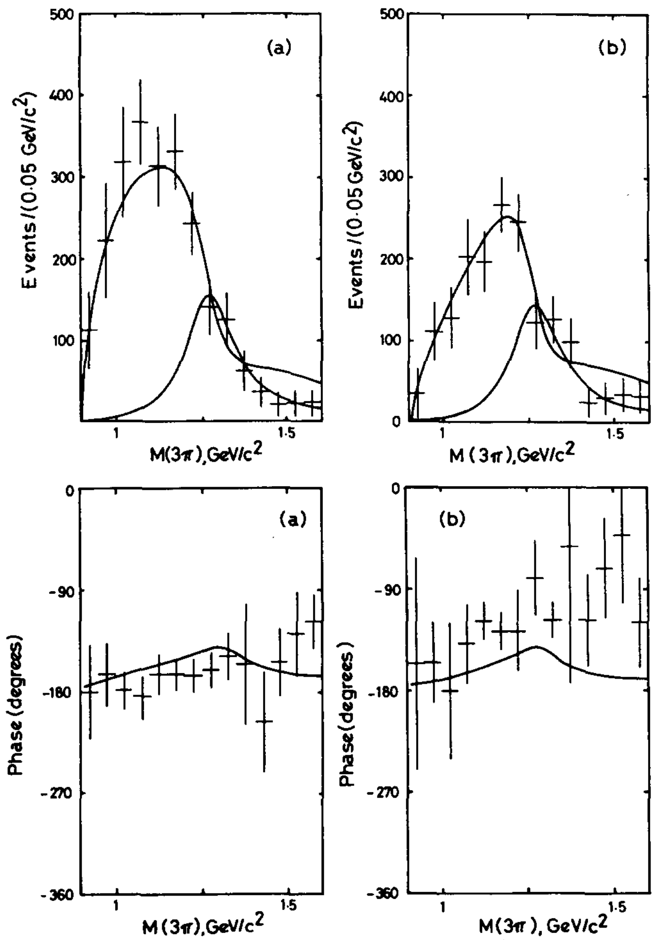

Fig. 7. The 1^+S intensity and phase with respect to 0^-S for (a) $0.0 < |t'| < 0.1$ (b) $0.1 < |t'| < 0.6$ (GeV/c)^2, the smooth lines are the result of a simultaneous fit to both t' regions, and the corresponding contribution from the pure resonance part.

variation across the peak is more pronounced. If the two regions are fitted independently, excellent fits are obtained with final parameters close to those quoted in ref. [4] but with τ/p_0 and χ two standard deviations on opposite sides of the value needed to fit the comprehensive t' region $0 \leqslant |t'| \leqslant 0.5$ (GeV/c)^2.

The Bowler formalism is not inconsistent with this variation, as the Deck amplitudes it requires for the 1^+S and 0^-S states are known to be t-dependent. To account for the 1^+S intensity variation a t' dependence of the form $e^{At'}$ may be included. A is a function of the 3π mass, assumed to be the t' slope determined experimentally on the entire data sample, dominated as it is by the 1^+S wave in this region. This extension of the model is motivated by the observation that for all components of the dynamical mechanism there is the same pomeron coupling at the proton vertex.

The values obtained with this modification of the model in a simultaneous fit to the low and high t' regions are then within errors of those already discussed for the comprehensive t' region, thus showing that the exponential t' dependence correctly accounts for the shifting 3π mass peak. The result of this fit is compared with the data in fig. 7a and b; while the mass distributions are in excellent agreement with the model, the fit is less satisfactory for the high t' phase. This however may be attributed to the use in the model of the 1^+S and 0^-S phases obtained by Ascoli et al. [5] at $t' = -0.1$ (GeV/c)^2. The use of phases appropriate to the t' bin analysed could then remove this discrepancy.

In conclusion, the data of this experiment indicate that the model constitutes a good parametrization of the observed 1^+S wave behaviour, the t' dependence observed being adequately described with a simple extension. It has been observed however, that in charge exchange data there is no evidence for an A_1 resonance of width $\lesssim 300$ MeV/c^2; thus the A_1 controversy is far from being resolved.

8. The A_2 resonance

Of all the structures in the three-pion partial-wave analyses the only contributing member of the natural J^P series is the only one which displays unambiguous behaviour. A relativistic Breit-Wigner fit to the data points of the $2^+D_{M=1}$ intensity is shown in fig. 8 assuming no other background. The resulting mass, width and cross-section parameters are given in table 5 with the value also of contributing unnatural parity exchange.

In fig. 9 an Argand diagram has been constructed using as amplitude the square root of the number of $2^+D_{M=1}$ events and as phase that of the off-diagonal density matrix element between this amplitude and that of 1^+S, presumed constant. There is a high degree of coherence between these waves ($\alpha = 0.90 \pm 0.13$) and thus every reason to believe that this phase represents the true phase difference between a dominating pair of amplitudes. The expected phase change of $\frac{1}{2}\pi$ for a full width across the resonance peak is clearly visible.

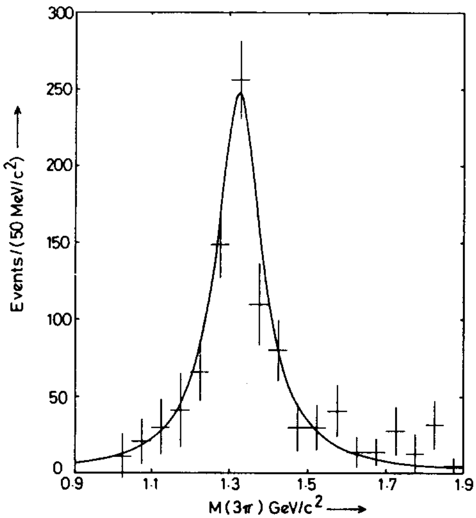

Fig. 8. The $2^+D_{M=1}$ intensity with a relativistic Breit-Wigner fit.

Table 5
Resonance parameters

	Mass (GeV/c^2)	Width (GeV/c^2)	Cross section[a)] (μb)	χ^2/NDF
A_1 (1^+S)	1.122 ± 0.008	0.267 ± 0.020	216 ± 45	30/18
A_2($2^+D_{M=1}$)	1.321 ± 0.006	0.140 ± 0.019		14/16
Natural parity exchange, $\eta = +1$			44 ± 10	
Unnatural parity exchange, $\eta = -1$ (M = 0, 1, 2)			2.3 ± 2.4	
A_3(2^-S)	1.607 ± 0.015	0.321 ± 0.055	64 ± 14	5.8/8
"A_4" ($2^+P_{M=1}$)	1.662 ± 0.006	0.024 ± 0.009	5 ± 2	5/8
"A_5" ($2^+P_{M=1}$)	1.796 ± 0.020	0.080 ± 0.065	7 ± 2	3.5/8
"A_4" ($2^+P_{M=1}$)	1.730 (fixed)	0.158 ± 0.019	13 ± 4	77/18

a) Normalized to a total reaction cross section 1.17 ± 0.24 mb [13]. All quoted cross sections are for integrated Breit-Wigners.

The differential cross section for the A_2 may be derived by taking a 200 MeV/c^2 wide mass bin and within this area parametrising the mass dependence of each partial wave according to results already mentioned, i.e. a Breit-Wigner parametrisation for the 1^+S and 2^+D intensity and roughly constant for the other waves. Fits may then be performed in small t' bins and the resulting $d\sigma/dt'$ of fig. 10 built up. This distribution favours a $t'\, e^{At'}$ dependence by a χ^2/NDF of 6.4/5 to that of a pure exponential

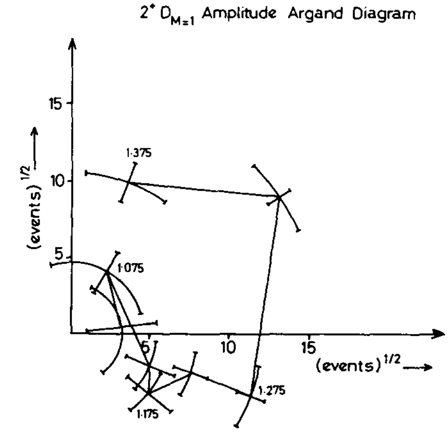

Fig. 9. Argand diagram of $2^+D_{M=1}$ amplitude.

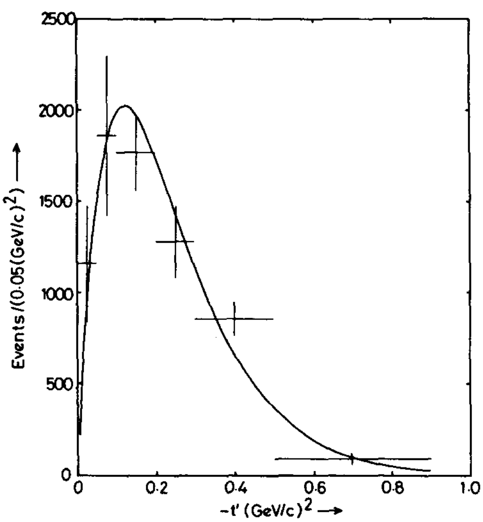

Fig. 10. Differential cross section for A_2, $2^+D_{M=1}$ intensity.

(χ^2/NDF = 40.5/5) indicating a net helicity flip of 1 for the reaction. The fitted slope is $A = -8.30^{+0.41}_{-0.44}$, to be compared with -8.6 ± 1.2 at 40 GeV/c [8] indicating no shrinkage.

In table 6 is shown the density matrix elements for the sub-matrix of the A_2 2^+D wave divided into combinations corresponding to natural and unnatural ($\eta = \pm 1$) exchange, for the mass interval $1.2 < M(3\pi) < 1.4$ GeV/c^2.

Table 6
Density matrix elements for A_2

Natural parity exchange	
$\rho_{11} + \rho_{1,-1}$	$= 0.85 \pm 0.08$
$\rho_{12} - \rho_{1,-2}$	$= (0.06 \pm 0.05) + i\,(-0.07 \pm 0.22)$
$\rho_{22} - \rho_{2,-2}$	$= 0.03 \pm 0.07$
Unnatural parity exchange	
ρ_{00}	$= 0.07 \pm 0.07$
ρ_{01}	$= (-0.03 \pm 0.05) + i\,(-0.01 \pm 0.21)$
$\rho_{11} - \rho_{1,-1}$	$= 0.02 \pm 0.07$
ρ_{02}	$= (0.0 \pm 0.05) + i\,(0.0 \pm 0.15)$
$\rho_{12} + \rho_{1,-2}$	$= (0.01 \pm 0.06) + i\,(-0.01 \pm 0.16)$
$\rho_{22} + \rho_{2,-2}$	$= 0.03 \pm 0.08$

9. The A_3 Region

It is clear from figs. 5 and 6 that the largest contributions in the mass region above 1.5 GeV/c^2 come from the 2^- partial waves, and of these only the 2^-S ($f\pi$) is large enough to account for the enhancement in the three-pion mass spectrum. The Breit-Wigner, with zero background, is illustrated in fig. 11, and the relevant parameters are given in table 5. The fit has a χ^2/NDF of 508/8 but a resonance interpretation has been rejected by a large statistics $(3\pi)^-$ analysis because of the lack of relative phase change between 2^-S and the background waves [8]. Two subsequent $(3\pi)^+$

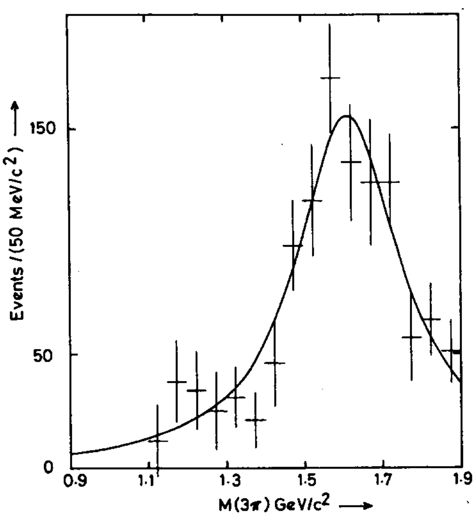

Fig. 11. The 2^-S ($f\pi$) intensity with a relativistic Breit-Wigner fit.

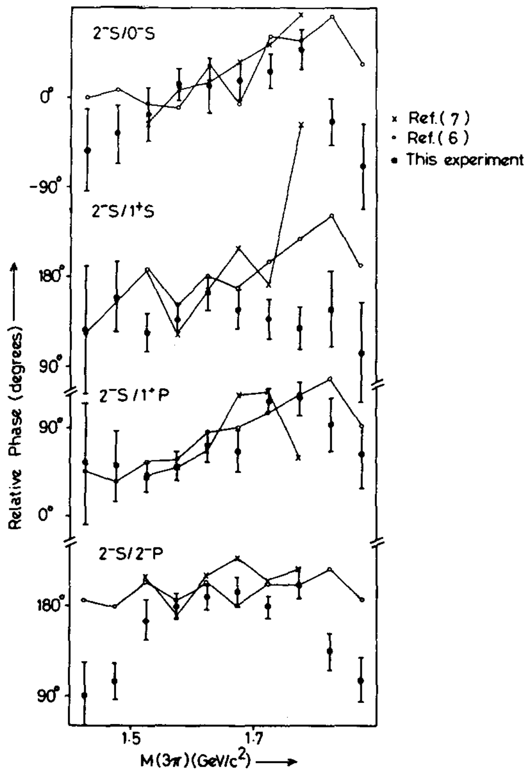

Fig. 12. Relative phases between 2⁻S and other dominant waves in the region of the A_3 enhancement.

analyses [6, 7] did, however, report phase changes suggesting a possible distinction between π^- and π^+ data.

The relative phases of the off-diagonal matrix elements are displayed in fig. 12 together with the results from the π^+ experiments. It may be observed that all data points are perfectly consistent and a net positive change of phase is suggested. It could still be, however, that a mechanism in the (fπ) system similar to that in the ($\rho\pi$) system in the A_1 region is occuring and that this phase change is due to a pure Deck-type effect or a Deck scheme which includes some genuine resonance production.

10. Further structure at high mass

A possible resonance phenomenon in the $2^+P_{M=1}$ (fπ) partial wave was quoted at a mass ~1.75 GeV/c^2 in an analysis [8] at 40 GeV/c in the interval $0.04 < |t| < 0.33$

(GeV/c)^2. This analysis was performed in 100 MeV/c^2 wide bins and quoted a width $\Gamma \sim 0.2$ GeV/c^2. The phase change relative to 2^-P, 1^+P and 1^+S waves was certainly great enough to permit a resonance interpretation. The analysis of the $(3\pi)^+$ system at 13.2 GeV/c [14] with $0.0 \leqslant |t'| \leqslant 0.5$ [14] in 50 MeV/c^2 intervals shows a similar behaviour in intensity with perhaps some indication of a steeper drop on the high mass side. The amplitude analysis at 7 GeV/c [12] with $0.1 \leqslant |t| \leqslant 0.6$ displays a similar rise, peaking at about 1.6 GeV/c^2 and then falling at the last data point analysed at 1.75 GeV/c^2. There is thus the possibility that there exist at least two enhancements in this partial wave.

In the hope of adding to information about such possible structures, the results of going to a 3π mass bin width of only 20 MeV/c^2 are displayed together with the Purdue data [14] in fig. 13. The error bars are large but it should be remarked that they are those of the usual energy-independent analysis, thus including an approximately constant systematic error associated with the overlap of the 2^+P state with the other partial waves. It may be seen that the hypothesis of structure within the general enhancement is supported with this new data. As is indicated in table 5, two narrow Breit-Wigners are much preferred over one broad one in fits to the intensity. It is also noted that errors on the phase determinations decrease with a decrease in the width of the bins used, indicative of rapid variations, but unfortunately present statistics and small interference with other waves still make these errors too large ($\sim 40^\circ$) to permit a firm conclusion.

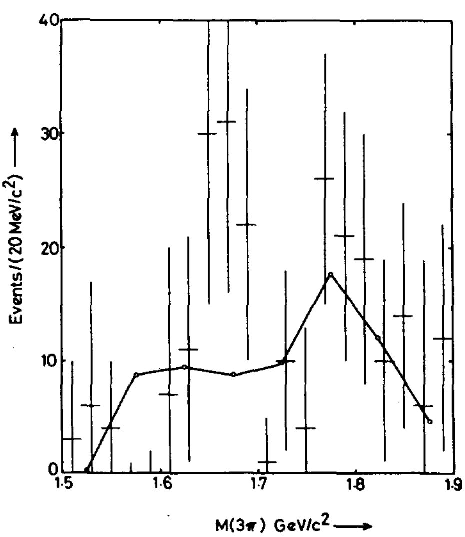

Fig. 13. Intensity of $2^+P_{M=1}$ wave. The solid line is the result at 13.2 GeV/c [14], normalised only to the overall total of events in this region.

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